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SOCIAL VALUATION OF PROJECTS: HARBERGER'S SOCIAL DISCOUNT RATE & THE PRICING OF RISKY PROJECTS.

by THIERRY F. BOLLIER

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comments welcomed

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ABSTRACT

In the presence of taxation of private returns, should the government attach to the gross cash-flows of a project the same value that the private sector would have computed when valuing the net-of-tax cash-flows?

One approach to this issue which has gained considerable following is Harberger's Social Discount Rate (SDR) Theory. In order to account for the alleged tax distortions, Harberger and others propose the use of a weighted average of observable market rates.

The rule is controversial in its treatment of fiscal effects and totally ignores risk.

The purpose of this paper is to develop a simple valuation rule that takes into account both the tax distortions and project-risk considerations. The main result is that tax distortions might lead a hurdle rate for certainty equivalent projects which differs from the market rate of interest, but that in general the project risk adjustments are independent of the tax distortions.

This rule is similar to the one proposed by Schmalensee[1976], but contradicts the highly referenced solution by Bailey and Jensen[1972].

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SOCIAL VALUATION OF PROJECTS: HARBERGER'S SOCIAL DISCOUNT RATE AND THE PRICING OF RISKY PROJECTS.

I. INTRODUCTION

> "For decades the most controversial issue in Cost-Benefit Analysis has been the selection of the appropriate Rate of Discount. The controversy initially focused on the relative merits of two candidate rates : (1) the (relatively low) social rate of time preference, which we shall call the consumption rate of interest (r) and which many writers have identified with the after-tax rate of return on private savings, and (2) the (relatively high) gross-of-tax rate of return to privately financed investment, which we shall call the investment rate (g). Recently, however, a number of analysts [...] have demonstrated that the Social Rate of Discount (w) should be a weighted average of g and r. As a result, the debate has been slightly refocussed, the candidate rates now being r and the weighted average w."
> L.Sjasstad and D.Wisecarver[1977], 513.

The theoretical literature dealing with public project valuation attempts to characterize what distinguishes public from private assets and/or what makes the government a special investor when acting on the behalf of the entire community. The list of controversies is extensive. The purpose of the current paper is neither to settle all of them, nor to propose a definite framework for correctly pricing public assets. Rather the focus will be on one approach, Harberger's Social Discount Rate (SDR) theory, as characterized in the above quotation from

Sjasstad and Wisecarver. This theory has gained considerable following in the cost-benefit literature along with recognition and extensive application in government circles[1].

Still, controversy remains on two specific grounds:

1) the legitimacy of Harberger's alleged capital market externalities, which rationalize the computation of w.

2) the treatment, in the public valuation of projects, of these externalities consistent with the accepted valuation procedures for risky assets.

The treatment of uncertainty in a SDR framework is subject to the greatest deal of confusion. The theory developed by Harberger[1969/72], Sandmo and Dreze[1971], and Dreze[1974] has traditionaly neglected risk considerations[2]. In practice a key premise seems to be that the sole reason for a wedge between the gross-of-tax returns in the private production sector and the net-of-tax market rate of time preference is a tax distortion. A lengthy argument has been raging on among Canadian academics (see Jenkins[1972],[1977], Burgess[1981], Hood,Glenday and Evans[1981],[1982]) to settle the question on the estimation of <u>the</u> Canadian discount rate; i.e. a rate that is used to price any public asset independently of its risk[3].

Whether uncertainty should justify price adjustments from a public perspective in an <u>undistorted</u> economy has also played a major role in the debate about social rates of discount. One extreme view was developed by Arrow and Lind[1970], who point to the large size of the government-owned portfolio and present an argument of perfect diversification. Hirschleifer and Shapiro[1970] and Sandmo[1972] show that Arrow and Lind's result is only a special case concerning the relationship

between private and public investments, and in general risk considerations should enter the calculations. However the discussion never integrates the results developed in the certainty case for the tax distortions and Harberger's SDR.

One remarkable attempt to approach the issue is the highly referenced article by M.Bailey and M.Jensen[1972]. Although most of the article was aimed at contradicting Arrow and Lind's[1970] argument of perfect government-owned-asset diversification, and thus centered on the relative abilities of the capital market and the government to share risk, they present as an example of desired risk adjustments, a Social Capital Asset Pricing Model which incorporates Harberger's weighted average solution. Unfortunately they do not provide a proof or derivation of their result.

The purpose of the current paper is to derive a simple valuation rule that takes into account both the tax distortions and project-risk considerations. The analysis will show that Bailey and Jensen's solution is wrong. The main result is that the tax distortions might lead to define a hurdle rate for certainty equivalent projects which differs from the market rate of interest, but that in general the project risk adjustments are independent of the tax distortions.

This solution is similar to the one proposed by Schmalensee[1976]. The main difference is that the current paper proposes a very simple derivation of the result which relies on both a version of Fischer Black[1981] Simple Discounting Rule to characterize the tax externalities, and an Adjusted Present Value Rule to account for these in public project valuations.

The following sections will attempt to deal with the issue

of public project valuation at three levels:

1) What question(s) is being asked when the SDR is used?

2) How can it be answered correctly?

3) How can it be answered most usefully?

Many other aspects of public finance models will be assumed away here. (For an excellent review of those issues we encourage the reader to refer to the recent book edited by R.Lind[1982].)

In consequence the paper is organized as follows. In Section 2 I discuss the current literature on Harberger-type social discount rates. The assumptions necessary for the issues to become neither trivial nor untractable are also stressed. Results leading to Harberger's weighted average discount rate are summarized at the end of the section. The simple valuation rule is developed in Section 3 and Section 4 shows how Bailey and Jensen's social CAPM fails. In the concluding section the main features of the simple valuation rule are summarized and it is shown that the same valuation rule could be applied to more complex environments.

II. SOCIAL DISCOUNT RATE THEORIES.

A. Harberger's displacements

The motivation for an SDR is rooted in empirical observations: the marginal before-tax-return on investment (the 'social' return or marginal rate of transformation in the economy) is larger than the after-tax required rates of return on private savings (the investors' risk adjusted rate of time preference or, unambiguously in the certainty case, the consumption rate of time preference). This fact is disturbing since apparently, if we were to set the social rate of time preference equal to the investor's equilibrium rate of time preference, rational investment policy should induce the government to undertake all available investments with marginal return higher than the common rate of time preference.[4]

However the government should also take into account the loss in future benefits due to the crowding out of private investments. From a social perspective the opportunity cost of private investment is the risk adjusted gross-of-tax rate of return in the private sector: if the effects of government investment in a given industry were to crowd out the same amount of the private competing industry, then there would be no incentive for the government to undertake a project whose return is lower than the private marginal rate of transformation. Hence the marginal rate of transformation, not the social rate of time preference, should become the hurdle rate to be used in discounting future costs and benefits from public projects.

SDR theorists point out that this line of reasoning holds only in the limit where the elasticity of private sector investments per dollar of public investment is identically equal to minus one. Their contention is that this is not necessarily true in a second best world where the return on private capital is taxed and a wedge is created between consumption rates of interest and marginal rate of transformation.

That the rules for optimal allocation of resources in an undistorted economy might not apply to distorted markets has been very recently reiterated by Robert Lind[1982], in his introductory chapter of a symposium on social discount rates: "The problem is that the corporate income tax and the personal income tax distort incentives.... Therefore, depending on where the resources for the public investment are drawn, the criterion for acceptance differs and, more specifically, the discount rate in one case should be the social rate of time preference, and in the other case, it should be the marginal rate of return in the private sector ".

Allegedly any amount invested (B) by the government is drawn either from new private savings (S) and/or from displaced private investments (I) (crowded out). This is formalized for the marginal dollar invested in the following equation :

$$(2.0) \qquad 1 = \frac{dS}{dB} - \frac{dI}{dB}$$

The social discount rate is then a "weighted average of the private sector marginal productivity of capital [i.e. the before tax return on capital] and the net-of-tax yield on private savings ..[which is a]... measure of the marginal rate of time preference in the private sector"[Harberger 1969,1972].

$$(2.1) \qquad w = i\,(1-T_i)\frac{dS}{dB} - i\,(1/(1-T_c))\,\frac{dI}{dB}$$

where:
i is the observed (riskless) market rate of interest
T_i is the marginal tax rate on savings
T_c is the marginal corporate tax rate.

Equation (2.0) is Harberger's fundamental displacement equation.

Detractors of Harberger's social discount rate argue against the theory on three specific grounds :

1) Equation (2.0) does not make much sense from the point of view of a social welfare maximizing central planner. The issue is similar to the one familiar from the optimal/neutral tax literature. In that context Diamond and Mirrlees[1971] and Auerbach[1982] in their respective analyses, recommend the use of discount rates for public investment decisions which equate the public and private marginal rate of transformation.

Aggregate production efficiency requires both that the government chooses an optimal level of equilibrium interest rate (at which the private sector will discount after tax project cash-flows) and that it designs its tax and incentives schemes such that the marginal rates of transformation are equal across productive activities. In such an equilibrium it can be shown that a unit of public investment displaces exactly a unit of private investment[5]. Hence optimal taxation would require (-dI/dB) to be identically equal to 1, and the right decision rule must assume this identity when defining the hurdle rate to be used in the public valuation. In this event the weighted average rate (2.1) boils down

being identically the private sector (gross) marginal productivity of capital, hence:

Proposition 1: except for cash flow externalities the public sector should value projects in the same way the private sector does, i.e. the present value calculation should be performed either on the total cash flows discounted at the before tax marginal rate of return on capital or equivalently on simulated after tax cash flows at the private sector discount rate.

The proposition is the only one acceptable in an undistorted (first best world) economy. Or it assumes that a central planner has the ability to enforce Diamond-Mirrlees standard of production efficiency. In either case the interest rate level (after tax) is chosen optimally, consumption levels are fixed and a unit of public investment must exactly displace one unit of private investment.

But Diamond and Mirrlees'[1971] result requires that all flows between the household and production sectors may be taxed. In fact this equilibrium will be socially desirable only to the extent that the central planner does have sufficient control of the necessary instruments. If the government cannot monitor continuously the instruments which would allow him to equalize the marginal rates of transformation and optimize the level of interest rate, and if a tax on capital income has created a wedge between the rate of time preference and the rate of transformation (before tax rates) in the economy, there does not seem to be a case for assuming such an identity ($-dI/dB=1$) in public valuations.

For the identity to hold we need both that the economy started from production efficiency (optimal interest rate and equal rates of transformation), and that the private sector anticipates that, for any

shift in equilibrium conditions, the government will take the necessary actions to bring back production efficiency in the economy[6].

In the distorted and non production-efficient economy the private investors equalize the after tax rate of return on capital with marginal rate of time preference. Hence the after tax rate of interest is effectively the shadow price for any marginal amount of forgone consumption. In other words the lower rate determines the investors' allocation between consumption and savings. The same investors would be better off if they were allowed to invest in any investment yielding more than the equilibrium after tax rate of interest. Except in the very unlikely event where consumption is inelastic with respect to the rate of interest, shifts in savings are bound to occur.

Hence a public project would tend to displace both private consumption and investments. If the central planner's objective is not to control for equalization of rates of transformation in the economy, a weighted average of the marginal rate of transformation and the lower marginal rate of time preference might better represent the opportunity cost of the public project. Since a dominant strategy for the public authority would be to eliminate distortions due to taxation and move to an undistorting set of transfers in order to achieve at least the same level of social utility, we must assume then that such an opportunity is not available, or for some unspecified reason actual taxation does not lead to production efficiency.

Under the alternative assumption that the government does not achieve production efficiency, Harberger's alleged displacements are not incompatible with general equilibrium, nor do they violate any assumptions on the maximization behavior of the private sector. The only

requirement is that the economy be distorted by the taxation of returns.

2) The second line of criticism, as stressed in modern finance theory, is that mistakes are often made in attempting to derive the opportunity cost of a given project from the apparent costs of explicit financing. For each particular project there might exist a project-specific cost of capital which can be best estimated by looking at similar competing projects rather than trying to trace the required rate of return from the different securities used for its financing.

The "natural" temptation for the financial economist is to argue that the marginal rate of return on private and public investments should be the same in all events. No matter what might be the impact of a public project on private consumption and investment the opportunity cost is represented by the marginal private investment : the government should never invest in projects with lower returns.

Proposition 1 can be rewritten as follows: <u>despite any actual effect on private consumption the public project should be valued at the before tax marginal rate of return on investment</u> (as if the elasticity of private investment was identically equal to one). Again the correct decision rule is obtained if one always assumes that the public project is crowding out the same amount of private investment.

This line of argument would surely be correct if the option to undertake the marginal private investment was open to the government. This might not be the case in reality : private sector businesses may have a competitive advantage over public enterprises for all sorts of reasons and the production function of the public sector might be very different from the private one. Other reasons could be outright bans on government

involvement in specific and numerous sectors of the economy, or the uninforceability of an efficient set of subsidies to the private sector.[7]

We shall assume in what follows that the private and public sectors choose their level of investment from two different sets of technologies[8].

3) A third line of argument, which Lind[1982] attributes to Harberger, goes as follows: public expenditure contributes to the budget deficit and must be financed through debt. The financing of the additional amount of debt will crowd out an equal amount of private investment (and again we would be lead to assume -dI/dB=1).

We must accept the fact that the tax system cannot be expected to be flexible enough to absorb any fluctuation in the level of public expenditures. Taxation can only be adjusted at discrete time intervals and the marginal dollar of public investment will always be financed out of government debt.

However the most convenient stand with respect to this issue is to adopt R.Barro's[1974] counterargument : agents are not fooled by the government's use of debt which is just a shift of current taxation for future taxation. The debt issue is fully transparent in that sense. The allocation between consumption and saving should be the one that would have prevailed if first period consumption had been taxed instead (i.e. the debt issue should not affect savings compared to the no-debt case and all additional funds are taken from consumption).

Before concluding this chapter on the rationale behind equation (2.0), we can make one last statement. It is true that if the

government action were not fully transparent in Barro's sense, the financing of the project could result in an additional increase in the interest rate which in turn would contribute to crowding out more private investments. On the other hand as pointed out in Lind[1982], the impact on private investments of the original public project and/or its financing might very well depend on whether or not resources are currently fully employed in the economy. He states that an increase in production of unemployed resources,if they exist, might very well match the increase in aggregate demand, and adjustments will be made at the expense of minor crowding out in the private sector. The level of savings could also be affected when the induced reallocation of capital favors a better utilization of factors of production (e.g., labor, clean air), although these effects would be inexistent or could be ignored if production efficiency was allowed, as in Diamond and Mirrlees[1971]. Eventually Lind concludes :

> First, one cannot reasonably divorce the analysis of public investments from macroeconomic considerations because the financing of public investments has different impacts on the economy depending on the state of the economy.... Second.... the choice of criteria by which to evaluate public investments in a second best world depends critically on how one beleives the economic and the political systems work....

In summary we can state a set of assumptions for the displacement equation (2.0). The same assumptions support including the alleged displacements in the analysis of the opportunity cost of public investment.

1) the economy is distorted by corporate and personal taxation of return and hence in equilibrium a wedge is created between the marginal rate of transformation in the economy and the marginal rate of time preference in the private sector.

2) the government does not achieve production efficiency and does not fully control the level of interest rate.

3) the government can neither undertake nor subsidize efficiently the marginal private sector investment and hence public and private sectors optimize over different production sets.

4) government borrowing activity does not absorb any savings in excess of the level that would have prevailed if the project had been directly financed through current taxation.

B. Review of results in the absence of project-risk considerations.

We have argued in the previous section that, given certain assumptions, Harberger's displacements equation (2.0) must be incorporated in any analysis of the opportunity cost of government investments. This equation is totally general, but we can expect the displacements of both saving and private investments to be closely related to the characteristics of the project, e.g., project total return, project life, type of resources used in production,etc.

Sandmo and Dreze[1971] and Dreze[1974] point out for the need of a detailed specific assessment of the "opportunity cost applicable to investments at a particular time and place". Nevertheless in estimating equation (2.1) most empirical studies have assumed that there are standard (non project-specific) adjustments of private savings and investments per dollar invested (see Jenkins[1972,1977], Burgess[1981], and Hood, Glenday and Evans[1981,1982]). The social discount rate (w) is "designed" to account for the "financing" externalities, independent of the specific project under consideration.

Although this assumption may be reasonable in the derivation of theoretical models aimed at studying the bias introduced by the displacements in the opportunity cost of a project, the simplification is often improper when the purpose is to derive a schedule of discount rates to be used in actual valuations. We shall come back to this issue later when we will discuss the possible implementation of the valuation framework in real life cases. For now we shall make the same assumption.

The main SDR result remains Harberger's[1969,1972] equation (2.1). Most of the more recent papers have proved Harberger's proposition in two period settings (Sandmo and Dreze[1971]) or in multiperiod settings

(Dreze[1974]) or reconciled previous approaches with the weighted average solution (Sjaastad and Wisecarver[1977])[9].

In the certainty case the social opportunity cost of capital, w, can be rewritten in a notation which has become standard in the literature[10] :

$$(2.1)' \qquad w = r\ dS/dB - g\ dI/dB$$

where r is the after-tax indirectly observed rate of time preference, and g is the before-corporate-tax rate, or indirectly observed marginal productivity of capital.

The expected signs of dS/dB and dI/dB are positive and negative respectively. These terms express the net impact of an increment of government borrowing, B, upon private savings, S, and private investment, I.

Harberger argues that the model can be generalized to a multi-market risky economy, where required rates of return and tax rates can differ across classes of investors, investments and securities. He derives the "general" formula for w :

$$(2.2) \qquad w = \sum_k i_k.(1 - T_{ik})\ dS_k/dB - \sum_j i_j/(1 - T_{cj})\ dI_j/dB$$

where T_{ik} is the marginal rate of personal income tax applicable to the kth class of savings, and T_{cj} is the marginal rate of corporate income tax applicable to the jth class of investments.

In (2.2) displacements are summed over all possible sub-markets (enough sub-markets are defined for each to be homogeneous in terms of required returns and marginal tax-rates),

In the standard notation,

$$(2.3) \qquad w = \sum_k r_k \, dS_k/dB - \sum_j g_j \, dI_j/dB$$

The partitioning is justified intuitively because investment sub-markets could be thought of as different industrial sectors with different systematic risks, different cost of capital and different marginal tax rates on income (the government might set different tax-transfer rules) and, equivalently, because different income-class citizens face different marginal tax rates on their security holdings.

Harberger[1969,1972] correctly notes that each "i_j [and i_k] obviously incorporates a premium for risk and whatever other factors which influence portfolio decisions. They can be represented as $i_j = i_f + b_j$, where b_j is the (quite possibly variable) "risk premium" required for investment in the jth activity and i_f is the (risk-free) rate of interest on government bonds".

We shall not review SDR theories any longer, except to stress that in practice social discounters intend to perform two related exercises with Harberger's social discount rate :

1) The measurement of w based on estimated elasticities of domestic investments, elasticities of displaced consumption and the incremental foreign financing[11].

2) Although not always stated explicitly, they use w as the

sole discount rate for any public project valuation, independent of its risk.

Although w in (2.3) corresponds to a weighted average cost of capital for the national portfolio, it is inappropriate to use it to allocate resources among the assets of this portfolio when potentially each project contributes differently to the total risk. This is a fundamental principle of financial economics.

As Harberger recognizes it, the proposed w incorporates the premiums for risk of each activity; hence w calculated from (2.3) could neither be viewed as the certainty equivalent rate that could be used to discount correctly the certainty equivalent cash-flows of the projects, nor as a risk adjusted discount rate. The analysis that follows should demonstrate these points.

Before proceeding further let us state an additional warning. Whether the valuation should account for the hypothesized displacements is a separate issue. We have presented in the current section a summary of the arguments against the production efficiency hypothesis and we will not extend on it any further. As pointed out by Auerbach[1981] though, to recognize that in principle the optimal tax schedule is likely to diverge from production efficiency, does not necessarily mean that we have other choices in practice since it might be "extremely difficult even in a very simple model to calculate merely the direction [of] this divergence ... ". Hence it might be extremily difficult in practice to estimate the displacements.

III. THE COSTS OF DISPLACEMENTS - AN ADJUSTED PRESENT VALUE APPROACH

Let me first restate the problem.

Under the conditions stated in Section 2 the public investment will be financed out of displaced savings and investments as described by Harberger (equation 2.0). How can we best account for these displacements in the public valuation of projects? The social discount rate theory has proposed the weighted average w, under the claim that it truly reflects the social opportunity cost of capital. Stiglitz[1982] reminds us that there is a trivial sense in which the value of a project can always be adjusted for potential externalities. This is simply: value all future costs and benefits by use of the marginal (social) rate of time preference. The public valuation differs from a private valuation to the extend that the total project cash-flows differ from the direct cash-flows, appropriable in the private sector. Because of the wedge between the rate of time preference and the rate of transformation cash-flows of the forgone private investments crowded out by the new project should be treated as part of the cost externalities. The total cash-flow approach would allow us to value sequentially the direct cash-flows of the project and the cost-benefit externalities; here the present value of the forgone investments.

Sjaastad and Wisecarver[1977] have argued that the two approaches are identical if consistent assumptions are made. In particular they were dealing with the case of certainty where no discrepancies in the rates entering w could be attributed to risk premia. In the case of uncertainty no simple adjustment could reproduce the weighted average solution. The total cost-benefits approach offers an alternative way to

study the problem that can be very easily accomodated in the framework familiar from the finance literature, known as the adjusted present value (APV) of a project. The only potential problem is to provide the appropriate value for the shadow price of forgone investments,

In what follows we propose the use of the alternative approach to solve the problem of the public valuation in the case of uncertainty.

We shall first derive the one period case. It is instructive to examine that simple problem in some detail. We will proceed in two steps :

A. We first introduce the adjusted present value approach in a world of certainty. We introduce the APV approach in this simple framework in part to show how the rule can be used to isolate projects which would have been rejected as part of the private sector opportunity set; i.e., public projects which display positive present values only because of the alleged displacements.

B. Second we derive a fundamental result about the value of forgone investments in a world of uncertainty. This result, along with a restatement of the APV rule, will form the basis of generalization to the case of risky projects, to the multi-period case and to more general valuation procedures for the underlying investment (real options, multi-beta CAPM economies), as briefly discussed in the conclusion of the paper.

A. Adjusted Present Value approach in a world of certainty:

Since we want to consider the effect of tax distortions on the valuation of public investments we introduce a tax on corporate returns

T_{cj}. This tax rate can vary across industries (j). For simplicity we assume as a first step that there are no personal taxes on savings. Hence in competitive equilibrium the private sector equates the expected after-tax-return on the marginal dollar invested in a project ($i_j(1-T_{cj})$) with the required rate of return on savings (r_j) for a comparable level of systematic risk.

Under these assumptions we want to determine the current net present value of a proposed single period project that will bring an end-of-period risky net cash inflow of C_1 and will require a current net cash outflow C_0. The project is to be undertaken by the government and, hence, is drawn from the public investment opportunity set. (We assume in particular that the marginal private investment are neither accessible to the public authority, nor can be efficiently subsidized.)

The rationale for the adjusted present value approach is to disregard any adjustment of the discount rate and value all the project-specific cash flows and cash flow externalities at the appropriate social discount rate. Arbitrarily we set the social rate of time preference to the after-tax-rate of return in the private sector, hence we will use this (lower) rate to perform the present value calculations.

In this simple case where the only cash flow externalities correspond to the alleged displacements, we can write:

$$(3.0) \qquad NAPV_{pub}(C_1) = NPV_b(C_1) + NPV_f(C_1)$$

$$= PV_b(C_1) - C_0 + C_0 - PV(H_1)$$

The first part of the public valuation ($NPV_b(C_1)=PV_b(C_1)-C_0$) is equal to the value of direct cash-flows from the project. The second part ($NPV_f(C_1) = C_0 - PV(H_1)$) corresponds to the present value of displaced future consumption (H_1) due to the displacements of savings and private sector investments. By analogy to the corporate sector environment we shall refer to this second term as the present value of the "financing" externalities. If the new project is to be financed entirely out of savings, the present value of forgone consumption when discounted at the marginal rate of time preference, would be exactly equal to C_0, the financing externality would be identically equal to zero and the public value of the project would be equal to the present value of the direct cash-flows. As we shall see below in general this will not be the case.

Since we are in a world of certainty, we have the base-case present value in the simple formulation :

$$(3.1) \quad PV_b(C_1) = \frac{C_1}{1 + r_f}$$

where, r_f is the riskless rate of interest, assumed to be equal to the social rate of time preference.

Given the sole taxation of corporate returns and equation (2.0) for the displacements we have the following expression for the <u>total amount of consumption</u> displaced (see below) by the new project next period (H_1) :

$$H_1 = \left[\sum_k (1+r_k) \, dS_k/dB - \sum_j (1+ r_j/(1-T_{cj})) \, dI_j/dB \right] . C_0$$

The investment side, i.e. the right-hand sum, represents the social opportunity cost, in terms of future forgone consumption, of the

amount of capital diverted from private sector production. As in social discount rate theories, it corresponds to the after tax return in the respective sectors plus the tax revenues; i.e., the marginal before-tax productivity (one plus the marginal rate of transformation) of diverted funds in their alternative private sector use. On the saving side, the left-hand sum corresponds to the social opportunity cost of incremental savings in terms of future cash-flows accruing to the investors; i.e., the marginal return on savings minus the incremental tax revenues, which is equal to the after-tax return on savings (one plus the marginal rate of time preference) in the private sector. (See Harberger[1969,1972] for a formal description of these displacements).

If the incremental project is small, the effect on equilibrium prices will be trivial, so that H_1 can be valued using marginal returns.

In equilibrium all market rates of return must equal r_f, hence we have:

$$(3.2) \quad H_1 = C_0 + \left[\sum_k r_f \, dS_k/dB - \sum_j r_f/(1-T_{cj}) \; dI_j/dB \right] C_0$$

since

$$\sum_k dS_k/dB - \sum_j dI_j/dB = 1$$

or we can rewrite H_1 by introducing an adjustment variable (T) which involves only the tax rates and the respective elasticities of saving and investment,

$$(3.3) \quad H_1 = C_0(1+r_f.T)$$

where

$$(3.4) \qquad T = \sum_k dS_k/dB - \sum_j (1/(1-T_{cj}))\, dI_j/dB$$

Notice that if the elasticities have the anticipated signs and at least one $T_{cj} > 0$, then $T > 1$, and in general $T \leq 1/(1-T_{cj})$ (if the tax structure is not too heterogeneous).

Hence in the certainty case the APV calculation collapses to a rather trivial exercise :

$$(3.5) \quad NAPV_{pub}(C_1) = \frac{C_1}{1+r_f} - C_0 + C_0 - \frac{C_0(1+r_fT)}{1+r_f}$$

We can notice that the decision rule is equivalent to the one advocated by social discount rate theorists, which states : use the social discount rate, r_fT, to value public project cash-flows.

$$NPV_{soc}(C_1) = \frac{C_1}{1+r_fT} - C_0 = \frac{C_1 - C_0(1+r_fT)}{1+r_fT}$$

Clearly $NAPV_{pub}$ and NPV_{soc} will always display the same signs, although not the same value. In any case the adjusted present value approach (4.5) does not complicate the valuation beyond the steps that were requested in the adjusted discount rate approach.

The remaining of the section on the certainty case will discuss several features of the APV approach. In particular we show how the valuation behaves under the assumption that public investments displace one for one private investments (the optimal-tax/production-efficiency

assumption) and how the rule can be used to isolate the projects which display a positive NPV only because of favorable hypothesized displacements of savings.

We can remember from the discussion in Section 2 that the link between the alleged displacements and the opportunity cost of public investment is the sensitive issue. Some would like to argue that $-dI_c/dB=1$ identically, so that r_fT collapses into $r_f/(1-T_{cc})$. Notice that here we refer to an investment class "c" such that the before-tax-discount rate does actually correspond to the marginal rate of transformation of the private investment crowded out; or we could assume that tax rates are equal across sectors which would be consistent with the production efficiency assumption; or that in any instance the marginal rate of transformation in the private sector is the only relevant opportunity cost (and equivalently the discount rate equals $r_f/(1-T_{cc})$). In other worlds these critics state what we will refer to, hereafter, as the neo-classical rule[12] or proposition 1 (Section 2) : take the public project if and only if

$$NPV_{nc}(C_1) = \frac{C_1}{1+r_f/(1-T_{cc})} - C_0 \geq 0$$

Although we have discussed at length the set of realistic assumptions which have lead us to reject in principle this extreme position, we would like to keep them as a benchmark in the public decision. We can adapt the valuation rule (3.4) and (3.5), in order to distinguish the proportion of the public value that would have been obtained by use of a neo-classical rule and the proportion that accrues because of the hypothesized shift in savings.

This can easily be done by transforming (3.4) and (3.5) [13]

$$(3.7)\quad T' = \sum_k dS_k/dB - \sum_j (1/(1-T_{cj}))\, dI_j/dB - (1/(1-T_{cc}))(dI_c/dB + 1)$$

$$= T - 1/(1-T_{cc})$$

==>

$$(3.8)\quad NAPV_{pub}(C_1) = \frac{C_1}{1+r_f} - C_0 + C_0 - \frac{C_0(1+r_f/1-T_{cc})}{1+r_f} - \frac{C_0\, r_f T'}{1+r_f}$$

Clearly if we were to leave out the last term, (3.8) would lead to the same decision rule as (3.6), the neo-classical rule; i.e., $NPV_{nc}(C_1)$ and $NAPV_{pub}(C_1) + C_0\, r_f T'/1+r_f$ would be both positive or negative simultaneously. Hence by analogy we rewrite :

$$(3.9)\quad NAPV_{pub}(C_1) = NAPV_{nc}(C_1) - PV(H'_1)$$

where,

$$NAPV_{nc}(C_1) = \frac{C_1}{1+r_f} - \frac{C_0(1+r_f/1-T_{cc})}{1+r_f}$$

$$H'_1 = C_0\, r_f T'$$

$NAPV_{nc}(C_1)$ and $NPV_{nc}(C_1)$ will in general have different magnitude since the denominators are not the same, but as discussed above the signs will always be identical.

Notice also that H'_1 will in general be negative. This is the case for two possible sets of reasons :

First, $-dI_j/dB < 1$ and a tax structure of corporate returns not too heterogeneous imply, as pointed out before, $1<T<1/1-T_{cc}$; hence

$T'<0$.

Second, even in the case where no saving is created (neo-classical assumption) but the taxation of corporate returns is disparate, and the public investment tends to crowd out low taxed private projects (relative to T_{cc}), T would tend to be close to 1 and hence T' might again be negative. However the converse can also be true if the public investment tends to crowd out highly taxed private projects.

In consequence we will tend to have two types of positive-net-present-value public projects:

1. Type one projects which display positive $NAPV_{pub}$ because both elements in (3.9) are positive; i.e., the project would have been accepted even if we were using the neo-classical assumption.

2. Type two projects which display positive $NAPV_{pub}$ only because the size of the externality (the present value of the alleged displacements in excess of the neo-classical assumption) is large enough to offset a negative net present value under the neo-classical assumption.

We might want to isolate type two projects because neither the theory nor empirical evidence can be expected to provide the decision maker with reliable estimates of the displacement equation(T or T'). The framework allows easily to pin-point such projects, to proceed to some sensitivity analysis of the estimate of T (or T'), and at the same time to keep as a benchmark the neo-classical valuation.

We can stress an additional feature of (3.9). Provided we reinterpret the last term we can compute the "bribe" (tax break in next period) necessary to get a private investor to undertake the socialy desirable project.

B. The costs of displacements in a world of uncertainty, and the valuation of risky projects:

I do not intend to restate the Bailey and Jensen[1972] argument showing why risk should enter into the calculation. Their article seems to have brought an end to the general acceptance of the use of the riskless rate in public valuations (at least on the grounds of the perfect diversification argument). Much of the discussion was centered on the relative abilities of the capital market and the government to share risk. Along the same lines Sandmo[1972] formally shows that in a stock market economy each consumer has unlimited possibilities for portfolio diversification, all risks priced in the market are true risks to society, the sources of which are to be found in the nature of the technology. Hence the government has no advantage over the individual consumer in this respect.

The most prominent feature of these approaches is not only that public discount rates should contain risk premia, but also that these should correspond to the one used in the private sector. This feature is to be found also in Schmalensee[1976] when he derives the result with complete insurance markets.

We shall share the same point of view and assume that the public and private sectors are identical in their assessment and pricing of risk. (For most of the following discussion, risk means the project's marginal contribution to the variance of the aggregate market portfolio; i.e., systematic risk in a CAPM sense, the covariance of future project cash-flows with the return on the market.)

.Public valuation under uncertainty:

As introduced in the certainty case we propose to value sequentially the total costs and benefits of the project, inclusive of the induced costs of forgone investments.

There is a trivial sense in which we can always apply a rule as developed in the previous chapter, once we apply it to certainty equivalent cash-flows. The main result in the next section is that the calculation of certainty equivalent displacements can also be simplified to a trivial exercise once we realize that, in equilibrium, certainty equivalent of risky market rates of return must be equal to the riskless rate.

Let us now restate our original problem in a risky environment.

In the one period setting the necessary conditions for the traditional Sharpe-Lintner capital asset pricing model (CAPM) are assumed to hold.

As before we want to consider the effect of tax distortions on the valuation of public investments. Hence we introduce a tax on corporate returns T_{cj}. Now this tax rate can vary across industries and risk classes (again for simplicity we assume as a first step that there are no personal taxes on savings). Hence in competitive equilibrium the private sector equates the expected after-tax-return on the marginal dollar invested in a project ($E_0 i_j (1-T_{cj})$) with the required rate of return on savings ($E_0 r_j$) for a comparable level of systematic risk.

Under these assumptions we want to determine the current net present value of a proposed single period project that will bring an end-of-period risky net cash inflow of C_1 and will require a current net

cash outflow C_0. As before the project is to be undertaken by the government and, hence, is drawn from the public investment opportunity set The public remains the residual claimant on the government assets and any excess return (or deficit) is transferred to(or taxed away from) end of period domestic consumption. Although there might be a distinct debt transaction or tax-transfer payments, the net position of the public is as if the government had issued equity on the new asset.

Again we set the social discount rate to be equal to the after-tax-rate of return in the private sector. Since we want to use a single riskless discount rate we must also adjust all cash flows for risk; i.e. derive the certainty equivalent cash flows, which is an easy task under the conditions of the one period CAPM.

Formaly we can write in our simple case where the only cash flow externalities correspond to the displacements :

$$(3.10) \quad NAPV_{pub}(C_1) = NPV_b(C_1) + NPV_f(C_1)$$
$$= PV_b(C_1) - C_0 + C_0 - PV(H_1)$$

where, NPV_b is the base case net present value of the direct cash flows from the public project,

NPV_f is the net present value of the 'financing' externalities, i.e. the present value of displaced future consumption (H_1) due to the alleged displacements of saving and private sector investments.

Since CAPM holds, we have the base-case present value in the following certainty equivalent formulation :

$$(3.11)\quad PV_b(C_1) = \frac{CE(C_1)}{1 + r_f} = \frac{E(C_1) - h.COV(C_1, r_M)}{1 + r_f}$$

where

h is the market price of risk
r_f is the riskless rate of interest
r_M is the after-tax-rate of return on the market.

Given the taxation of corporate returns and equation (2.0) for Harberger's displacements we have the following expression for the total amount displaced next period (H_1) :

$$H_1 = \left[\sum_k (1+r_k) \, dS_k/dB - \sum_j (1+ r_j/(1-T_{cj}) \, dI_j/dB \right] . C_0$$

Notice that the returns entering H_1 are random variables, so that H_1 is also a random variable. In order to assess the present value of expected displacements (H_1) we might want to calculate the covariance of H_1 with the return on the market portfolio. This might be difficult in practice. There is a much simpler way to value the uncertain displacements, which we introduce now.

H_1 simplifies to

$$(3.12)\quad H_1 = C_0 + \left[\sum_k r_k \, dS_k/dB - \sum_j r_j/(1-T_{cj}) \, dI_j/dB \right] C_0$$

since

$$\sum_k dS_k/dB - \sum_j dI_j/dB = 1$$

or we can rewrite H_1 in a form actually proposed in Harberger[1969,1972]

$$(3.13)\quad H_1 = C_0(1+r_f.T) + \left[\sum_k (r_k-r_f) \, dS_k/dB - \sum_j (r_j-r_f)/(1-T_{cj}) \, dI_j/dB \right] C_0$$

where,

$$(3.14)\quad T = \sum_k dS_k/dB - \sum_j (1/(1-T_{cj})) \, dI_j/dB$$

Notice again that if the elasticities have the anticipated signs and at least one $T_{cj} > 0$, then $T > 1$.

The derivation of the present value of the alleged displacements can be simplified if we recognize that the cash flows in the second term of the right hand side of (3.13) are a function of excess market returns and hence the present value of those returns must be equal to zero.

For a general discussion of this result see Fischer Black's[1981] Simple Discounting Rule. The reason, here, is actually quite obvious. Let us consider the present value of a particular cash flow : $A(r_c-r_f)$, where r_c is the return on an asset valued in the private market and in consequence satisfies the CAPM, and A is constant. The present value of this particular cash flow is

$$PV(A(r_c-r_f)) = \frac{A[(E(r_c)-r_f) - h.COV(r_c,r_M)]}{1 + r_f}$$

But since we can replace $E(r_c) = r_f + B_c(r_M-r_f) = r_f + h \ cov(r_c,r_M)$, we have :

$$PV(A(r_c-r_f)) = \frac{A[r_f + h.cov(r_c,r_M) - r_f - h.COV(r_c,r_M)]}{1 + r_f} = 0$$

Hence we have if CAPM holds

$$PV(H_1) = C_0 \ (1+r_f.T) \ / \ (1+r_f) \ , \text{ and}$$

$$(3.15) \quad NPV_f(C_1) = C_0 - PV(H_1) = C_0 \ r_f(1-T)/(1+r_f)$$

and the total value of the project :

$$NPV_{pub}(C_1) = \frac{CE(C_1) - C_0(1+r_fT)}{1+r_f}$$

One characteristic of this result is that the risk adjustment and the tax adjustment are independent of each other. Moreover the pricing of the risk is neither affected by taxation nor by the displacements of savings and investments; while the adjustment for the displacements could be performed exactly as in the certainty case by just using the riskless rate of interest and the tax variable T, and hence is independent of the risk premia embodied in the different returns on the investments displaced. This result agrees with Schmalensee[1976] but contradicts the highly referenced solution by Bailey and Jensen[1972]. As we shall see in part IV, their solution proposes the use of the variable T to adjust both the riskless rate and the risk premia, which is incompatible with our result.

If we consider the complete Harberger world with personal taxes on security returns we can derive a similar result. Now the observed rates in the market will not be the right measures for the riskless rate of time preference and its risk adjusted counter-parts.

If i_f, i_c are the market rates and we assume a flat personal tax rate T_i , then we have a CAPM equilibrium either in terms of observed market returns (i), or in terms of after-tax-returns $r=i(1-T_i)$ [14]

Then the base-case NPV is

$$NPV_b = [(E(C_1) - h.COV(C_1.r_M)) / (1+r_f)] - C_0$$

and the displacement effects are

$$H_1 = [\sum_k (1+ i_k(1-T_i)) dS_k/dB - \sum_j (1+ i_j/(1-T_{cc})) dI_j/dB] . C_0$$

Repeating the transformation

$$H_1=[\ 1+\ (i_f x T)\ +\ \sum_k (i_k-i_f)(1-T_i)\ dS_k/dB\ -\ \sum_j ((i_j-i_f)/(1-T_{cj}))\ dI_j/dB\ ]\ C_0$$

where (now adjusted for both personal and corporate taxes)

$$T\ =\ \sum_k (1-T_i)\ dS_k/dB\ -\ \sum_j (\ 1\ /(1-T_{cj}))\ dI_j/dB$$

and as before the present value of the excess returns equals zero. Hence

$$PV(H_1)\ =\ C_0\ .\ (\ 1+\ (i_f x T)\ )\ /\ (1+r_f)$$

and

$$NPV_f\ =\ C_0\ .\ (\ 1\ -\ (\ (1+(i_f.T))\ /\ (1+r_f)\ )\ )$$

(3.16) $$NAPV_{pub}(C_1)\ =\ NPV_b\ +\ NPV_f$$

$$=\ [\ (\ E(C_1)\ -\ h.Cov(c_1,r_M)\ )\ /\ (1+r_f)\ ]\ -\ [\ C_0\ (\ 1\ +\ i_f T\ )\ /\ (1+r_f)\ ]$$

In short our net public adjusted present value equals the base-case NPV plus the present value of future taxes displaced (computed as if all returns were equal to the riskless rate r_f and discounted back at the riskless rate (i.e. Fischer Black's simple discounting rule).

As in the certainty case we can transform the general solution into an equation that separates the neo-classical valuation and the value of the displacements in excess of the neo-classical assumption ($-dI_c/dB = 1$) :

$$(3.17)\quad NAPV_{pub}(c_1) = NPV_b(C_1) + C_0\, i_f(1-Ti-T)/(1+r_f)$$

<==>

$$(3.18)\quad NAPV_{pub}(C_1) = NAPV_{nc}(C_1) - C_0\, r_f T'/(1+r_f)$$

$$= \frac{CE(C_1)}{1+r_f} - C_0 + C_0 - \frac{C_0(1+i_f/1-T_{cc})}{1+r_f} - C_0\,\frac{r_f T'}{1+r_f}$$

where the simple discounting rule has been used once more in the calculation of $NAPV_{nc}(C_1)$; and T' is still equal to $T - 1/(1-T_{cc})$.

Equivalently equation (3.16) or (3.18) should be viewed as the general solution to public project valuation proposed in the paper. This remains a fairly simple valuation rule.

In summary the public valuation could involve two steps. As in equation (3.16) the base case NPV can be calculated. This involves the project direct cash-flows, the certainty equivalent operator and the social riskless rate of time preference. This first step is independent of tax distortions and displacements. Then in a second step is computed the net present value of savings and investments displaced in the economy. This net present value (NPV_f) involves only the social riskless rate of time preference, the market riskless rate of interest and the estimated tax/displacement variable T. Because of the tax distortions NPV_f will be equal to zero in the single case where only savings have been displaced. Because the polar case where only investments are displaced is an important benchmark since it corresponds to the optimal-tax/production-efficiency

assumption, we have proposed the equivalent rule (3.18). This rule distinguishes itself by splitting the present value of induced displacements between a first term which corresponds to the production efficiency assumption and a second term which corresponds to the induced displacements in "excess" of the neo-classic assumption; i.e. the first four terms of (3.18) would lead to accept or reject projects on the same basis than the private sector, The public valuation involves a fifth term because it is assumed that the project might not have been financed solely out of crowded out private investments. Notice that in the NAPV approach additional cash-flows externalities could easily be handled by adding new terms to (3.16) or (3.18).

As pointed out in the certainty case the only hazardous calculation remains to estimate T (or T'). Unfortunately this must be extremely difficult. T depends on elasticities of savings and investments. But in any case the adjusted present value approach does not complicate the valuation beyond the steps that were required in the adjusted discount rate approach. The result demonstrated for the valuation of the alleged displacements, should stress that the elasticities need to be estimated for the different effective classes of marginal taxation, but need not be related to the different classes of risk-adjusted returns.

A last warning about the cost of displacements. The term (in 3.16) $i_{f.T}$ is in no way equal to the weighted average cost of capital (w) proposed by Harberger and in practice, generally estimated and used in public valuations. "w" is valued at the marginal return on assets in the economy and is inclusive of the risk premia!

Before reexamining Bailey and Jensen's solution in the light of our results let us give the multiperiod (perpetuity) solution, when only

the first outlay generates costly displacements: in every period the project generates an uncertain cash-flow with constant expected value (C), and the initial outlay is C_0, then

$$\begin{aligned} NAPV_{pub}(C) &= NPV_b + PV_F(C_0) \\ &= C/r_c - C_0 + C_0(1 - T/(1-T_i)) \\ &= C/r_c + C_0 \left(T/(1-T_i) \right) \end{aligned}$$

IV. SYSTEMATIC ERRORS IN BAILEY AND JENSEN'S APPROACH

Bailey and Jensen's Social Capital Asset Pricing Model (SCAPM):

Bailey and Jensen[1972] have correctly argued for considering risk in public project valuations. They conclude as we did in Section III, that a reasonable guess might be to assume that the degree of optimality with regard to risk sharing on public assets is comparable to the one achievable in a private stock market. With this assumption in mind we ought to use publicly traded securities to extract the most reliable information on the two prices necessary for adequate valuation of risky projects: investors' riskless rate of time preference and the price of risk.

In other words Bailey and Jensen want to restate the problem of Harberger's externalities in the following terms. Assume that government ownership does not distort or improve risk bearing, if the correct measure of the riskiness of a public project is its marginal contribution to the standard deviation of the market portfolio (as a proxy for individuals' true portfolio), i.e., its beta under the standard assumptions of the Sharpe-Lintner CAPM. Then we can derive a set of adjustments which account for the tax distortions in the public valuations. This is exactly what were the assumptions of our derivation.

In order to solve this problem they propose, without much explanation, the following Social Capital Asset Pricing Model, which, they claim, provides the appropriate social risk premiums (i.e. including externality adjustments) to apply to individual projects.

The market-determined equilibrium private expected rates of return can be described by the traditional CAPM relation :

(4.1) $E(i_k) = i_f + h \cdot Beta_k$ (private or market rates)

where

$E(i_k)$ is the expected return on asset k (after corporate taxes but before personal taxes)

i_f is the riskless rate

h is the market price of risk per unit of standard deviation = $(E(i_M) - i_f) / G_m$

i_M is the return on the market

G_m is the standard deviation of the market return

$Beta_k = Cov (i_k , i_M) / G_m$

Bailey and Jensen claim that it is possible to derive a CAPM-like relation for social discount rates :

(4.2) $E(w_k) = w_f + k \cdot Beta_k$ (Public Rates)

where,

$$(4.2.1) \quad w_f = i_f \left[\sum_j (1-T_{ij}).dS_j/dB - \sum_k 1/(1-T_{ck}).dI_k/dB \right] = i_f \; . \; T$$

$$(4.2.2) \quad w_k = i_k \left[\sum_j (1-T_{ij}).dS_j/dB - \sum_k 1/(1-T_{ck}).dI_k/dB \right] = i_k \; . \; T$$

$$(4.2.3) \quad k = h \left[\sum_j (1-T_{ij}).dS_j/dB - \sum_k 1/(1-T_{ck}).dI_k/dB \right] = h \; . \; T$$

Notice that as pointed out in the previous section, the displacement equation (2.0) implies that each marginal personal tax rate will tend to drive T below 1, and each marginal corporate tax rate will tend to drive T above 1 (as long dS_j/dB and dI_k/dB have the expected signs). The net effect is uncertain:

$$T \quad \begin{matrix} > \\ = \\ < \end{matrix} \quad 1$$

It can go either way depending on the relative effects of government borrowing on savings and investments along with the relative size of the tax distortions.

No proof is provided, but at first glance the result is surprising. The elasticities are no longer used to compute a weighted average return; instead the constant term (T) multiplies both the risk-free return and the risk premiums!

The adjustment of the price of risk for tax distortions can hardly find a theoretical justification. If it happens that T is less than 1, why should the government apply to the valuation of the projects lower risk premiums than those which private investors would apply?

On the other hand none of the equations in (4.2) proceed from the same rationale as Harberger's weighted average w (in 2.2). The new version suffers from clear inconsistancies that were absent in the riskless version[15].

The Bailey and Jensen formula does not provide the right decision rule for risk adjusted public valuations, as formaly derived in the previous section. Ultimately we shall see that the error made in the social CAPM is very similar to that in the Modigliani-Miller adjusted cost of capital.

systematic errors

The costs of the alleged displacements on the original outlay of the government project is very similar to the adjustment performed in private project valuations to account for interest tax-shields on the project's financing. We can now anticipate that Bailey and Jensen did not actually derive a Social CAPM but rather performed an adjustment on risky discount rates very much like Modigliani and Miller Adjusted Cost of Capital. We can notice the similarity between the two formulations:

1. Modigliani and Miller cost of Capital:

$$i_c^* = i_c \left(1 - T_{cc} L_c \right)$$

where

i_c is the risk adjusted cost of capital on an all-equity firm

T_{cc} is (as before) the marginal corporate tax-rate

L_c is the project marginal contribution to the firm's debt capacity

2. Bailey and Jensen's social cost of capital

$$w_c = i_c \cdot T$$

As it is well known those adjustments lead to inexact decision rules for finite lived projects (see Myers[1974]), but the decision rules become exact for perpetuities. Nevertheless in both cases the adjusted cost of capital will lead to the wrong net present value calculations except when the net present value equals exactly zero. In other words, even in the perpetuity case the adjusted cost of capital just defines a hurdle rate, and does not necessarily give the right NPV. This is potentially more worrisome for the kind of decisions to be undertaken since often the government might consider resources for two mutualy exclusive projects of very different kinds and for which we might expect different induced displacements (and hence different T). Both valuations might display positive NPVs but the magnitudes of the calculated NPVs will be wrong.

The adjustment could be rescued (providing the right decision rule but still the wrong NPV calculations) in the finite-lived case if performed with riskless rate of return on certainty equivalent valuations. This also would apply to Modigliani and Miller adjusted cost of capital.

In order to see these results, let us consider the one period project of Section III.

Bailey and Jensen propose the use of :

$$W_c = i_c.T = (r_c/(1-T_i)).T$$

Hence they compute the following net present value :

$$(4.3)\quad NPV_{B\&J}(C_1) = \frac{E(C_1)}{1+r_c(1/1-T_i).T} - C_0 = \frac{E(C_1)}{1+i_c.T} - C_0$$

$$= \frac{E(C_1)}{1+[r_{f+}\ h.Cov(r_c.r_M)](1/1-T_i).T} - C_0$$

$$= \frac{E(C_1) - C_0\{1+[r_{f+}\ h.Cov(r_c.r_M)](1/1-T_i).T\}}{1+[r_{f+}\ h.Cov(r_c.r_M)](1/1-T_i).T}$$

The APV calculations in chapter III lead to the result (3.16)

$$NPV_{pub}(C_1) = \frac{CE(C_1) - C_0(1+i_fT)}{1+r_f}$$

$$= \frac{E(C_1) - h.Cov(C_1,r_M) - C_0(1+i_fT)}{1+r_f}$$

$$= \frac{E(C_1) - h.Cov(C_1,r_M) - C_0(1+r_f(1/1-T_i)T)}{1+r_f}$$

which can be rewritten

$$(4.4)\quad NPV_{pub}(C_1) = \frac{E(C_1) - h.C_0.Cov(i_{pc},r_M) - C_0(1+r_f(1/1-T_i)T)}{1+r_f}$$

where,

i_{pc} is defined as the project return (not necessarily equal to any specific market return).

Clearly (4.3) and (4.4) will not lead in general to the same decision rule, as long

$$Cov(i_{pc},r_M) = Cov(r_c,r_M).(1/1-T_i).T$$

No simple comparison between the two calculations can be made but in some cases Bailey and Jensen's approach will also give the wrong sign.

We can propose an alternative rule to Bailey and Jensen's adjusted discount rate by limiting the adjustment to the riskless rate : $w_f=i_fT$, and using w_f in certainty equivalent valuations.

$$(4.5)\quad NPV'_{B\&J}(C_1) = \frac{CE(C_1)}{1+w_f} - C_0$$

$$= \frac{CE(C_1) - C_0(1+i_fT)}{1+i_fT}$$

Clearly this new rule (which was the one proposed by Schmalensee[1976]) is consistent in sign with the adjusted present value approach (the numerator is the same as in (3.16)). That is, w_f defines the correct hurdle rate, although we still get an incorrect magnitude for the present value. The result carries through in very much the same way for Modigliani-Miller adjusted cost of capital if the adjustment were to be performed on the riskless rate and used only to discount certainty equivalents. The reason is that by proceeding in this fashion no

adjustment is, nor need be, performed on the price of risk. (Notice once more that w_f is not equal to SDR "w", and current estimates of w are of no use for the proposed modified adjustment rule)

Finally, in a multiperiod framework (perpetuities, see last equation of chapter III) :

$$NAPV_{pub}(C) = C/r_c + C_0(T/(1-T_i) = [C - C_0 i_c T]/r_c$$

but according to Bailey and Jensen's rule :

$$NPV_{B\&J}(C) = C/i_c T - C_0 = [C - C_0 i_c T]/i_c T$$

In both cases Bailey and Jensen's rule provides inexact net present values.

V GENERALIZATIONS AND CONCLUSION :

We have proposed in the paper a simple valuation rule for public investments.

The valuation rule can be summarized in the following way : the net present value of the project is equal to the base-case NPV plus the present value of displaced savings and investments, computed as if all returns were equal to the riskless rate and discounted back at the proper riskless discount rate. This reflects the application of Fischer Black's simple discounting rule. In this definition the proper riskless discount rate is the social rate of time preference, which we have set to be equal to the marginal rate of time preference in the private sector, although the result would carry through under reasonable departures from this assumption. However, the public sector must account for and price systematic risk in the same way as the private sector.

This simple valuation rule is appealing not only because it corrects the systematic errors made in the traditional adjusted social discount rate approach or in Bailey and Jensen's social CAPM, but also because it can be easily generalized to many valuation problems.

Valuation of projects cannot always be dealt with using discounted cash-flow rules. When the underlying real asset is a sequence of real options, only contingent claim analysis can help in valuing these projects. This is often the problem confronted in public valuations since many projects are energy related (oil projects are typically treated as a series of real options on future production) or are linked to explicit guarantees (the government commits itself to subsidies and bail outs that

would occur only in certain states of the world). Obviously adjusted discount rates can be of no help in adjusting for the hypothesized displacements, the valuation of such options.

The simple valuation rule holds in the case where the relation among market returns is better described by more general risk-return relationships, as in Merton's[1973] intertemporal asset pricing model, or Ross'[1975,1976] arbitrage pricing theory. In particular it can be shown that the last proposition is very general and that the present value of excess equilibrium returns is always equal to zero.

Also the paper along with the above discussion should lead to practical recommendations for the implementation of the valuation rule in practice. The APV distinguishes between the valuation of the basic project with the proper project risk considerations, and the valuation of the financing externality. The latter is proportional to the amount invested and is independent of the risk characteristics of the project (this is actually the main reason why Bailey and Jensen's rule is wrong) . The approach could also draw the attention to those projects that are only profitable because of favorable hypothesized shifts in savings.

FOOTNOTES

[1] Lessard, Bollier, Eckaus, Khan[1982] point out that although currently exists substantial controversy over the 'precise' estimation of the social discount rate "w" (the results range from 7 to 10 percent or more in real terms), the general approach has wide acceptance in Canadian public policy analyses.

[2] Some SDR theorists acknowledge the absence of uncertainty in their approaches but either they argue that the problem would justify a separate paper (Sandmo and Dreze[1971]), or they take for granted Bailey and Jensen's results (Sjaastad and Wisecarver[1977]), which as we shall see seem to accredit a mostly unmodified transposition of SDR adjustments in a multi-rate context.

[3] Glen Jenkins[1981] even argues in the case of the public valuation of an oil project against the use of the private opportunity cost of capital computed in the oil industry because the low return sectors such as "aircraft and parts, railways, water transport and grain elevator" brings the social opportunity cost of capital down and hence makes it a more reliable estimate of what would be the true opportunity cost of public funds. No mention is made of tax wedge here, only of discrepancies attributable to difference in risk premia!

[4] Other approaches than the one considered here often state that the social discount rate and social rate of time preference might have no links with market rates because cost-benefit analyses must take into account such externalities as: distribution externalities, poorly developed capital markets, absence of intergenerational bequest motives... On the other hand Harberger states that his theory is adapted only to the case of well developed economies with well functioning capital markets where these externalities can be left aside in the analysis.

[5] See Stiglitz[1982] p.165 for a simple derivation. Alternatively, if the economy is originally in a state of production efficiency, the private sector will not increase the level of saving when "offered" a public project with a return between the before-tax and after-tax rate of interest, and private sector investment will be crowded out more than one for one (I thank Fischer Black for pointing this out to me). This is the case because rational investors will anticipate that the loss of tax revenues will induce the government to raise taxes on future income. This increase in the tax rate must induce a decline in national wealth, otherwise we could not have been at a point of production efficiency initially. This contradicts the hypothesized shift in savings. In other words once the government has chosen the level of interest rate and has designed its taxes in consequence, consumption levels are fixed, and a unit of public investment must crowd out at the margin a unit of private investment.

[6] Once we relax the initial assumption of optimal control, we cannot obtain the strong result. If the economy was not in a production-efficient equilibrium in the first place and if the same experiment as in footnote 5 was performed, they might not even be a need for a shift in the tax rate for the government to protect its future tax revenues. There will also be

some uncertainty with regard to the behavior of the government. One's best guess might then be that the tax rates are fixed at the current level for all the relevant horizon. Under these circumstances public investment need not crowd out one for one private investments and equation (2.0) does offer a more general approach to the impact of public investment.

[7] This does not mean that all kind of subsidized investment is banned from such economies but rather that only an equilibrium is reached where none of the private sector with higher return than the project to be valued can be efficiently subsidized or publicly owned.

[8] Notice that this would not be a constraint under the conditions of Diamond and Mirrlees[1971] since a reduction in the rate of taxation is another way for the government to induce the undertaking of the marginal private project. The government could adjust the tax rate until production efficiency is achieved.

[9] Recently Arrow[1982], developing Arrow and Kurtz[1970], has looked at multiperiod growth models. They show that the opportunity cost of capital remains in these contexts a weighted average of consumption rate of time preference and marginal before-tax-return on private capital, but the weights would be very difficult to observe, project specific (related to the complementarity of private and public production) and related to other economic variables. However Sjaastad and Wisecarver[1977] argue that reinvestment of the net project output has negligible quantitative effects. They also propose to treat as "external" additional benefits to be attributed to the project, with the augmented benefit stream then being discounted by "w". Lind[1982] makes similar advice. This would also be very well adapted to our adjusted present value rule in the case of risky environments.

[10] An alternative version of w is also given by Harberger and more frequently used in SDR theories :

$$(2.1)' \quad w = (r\, dS/di - g\, dI/di) / (dS/di - dI/di)$$

or in terms of interest elasticities :

$$(2.1)'' \quad w = (r\, S\, e_S - g\, I\, n_I) / (S\, e_S - I\, n_I)$$

All those expressions are equivalent. The 'borrowing' version is simpler since if all markets are in equilibrium, $dS/dB - dI/dB = 1$. It is also the one used by Harberger when he discussed the possibility of different yields due to risk premia, and seems more intuitive in that context.

[11] In the small economy case, Harberger[1976a,1976b] (or see Hood, Glenday, Evans[1982]) introduces country risk in his model: an upward slopping cost of foreign funds drives the increase in the interest rate paid on savings and which feeds back in the crowding out of investments.

[12] We can notice that the neo-classical rule assumes more than that only private investments are crowded out, but makes additional assumptions on the homogeneity in terms of risk and taxation of the private projects displaced. Hence the displacement approach might be formulated even in the case where savings are not created in the process, but where we need to take into account the heterogeneity of taxation.

[13] I thank Stewart Myers for suggesting this formulation.

[14] This is a stronger condition than we actually need for a market CAPM to hold (on before personal tax returns). As discussed in Long[1977], what he calls the "efficiency equivalence hypothesis" can be met under fairly general description of the tax structure.

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